Math 241 Sample Problems for Midterm Exam

Question 1 Don't forget quadric surfaces.

Question 2 Let $f(x,y) = x\sin(y/x)$. Find the partial derivatives: $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial^2 f}{\partial y \partial x}$

Question 3 Find and sketch the domain of the function $f(x,y) = \dfrac{5}{\sqrt{10 - 2y^2 - x^2}}$.

Question 4 Let $w = f(u,v)$ be a function whose derivatives of all orders exist. Suppose that $\dfrac{\partial^2 f}{\partial u^2}(0,2) = 0$, $\dfrac{\partial^2 f}{\partial u^2}(3,0) = -3$, $\dfrac{\partial^2 f}{\partial u \partial v}(0,2) = 2$, $\dfrac{\partial^2 f}{\partial u \partial v}(3,0) = 3$, $\dfrac{\partial^2 f}{\partial v^2}(0,2) = 1$, $\dfrac{\partial^2 f}{\partial v^2}(3,0) = -1$. If $u = y + e^{2x}$ and $v = xy$, what is the value of $\dfrac{\partial^2 w}{\partial y^2}$ evaluated at the point $(x,y) = (0,2)$.

Question 5 Find the direction in which $f(x,y) = x^2 + \cos xy$ increases most rapidly at the point $(1, \pi/2)$. What is the rate at which f changes in that direction? What is the equation of the tangent plane at the point $(1, \pi/2)$?

Question 6 Find the critical points of the function

$$f(x,y) = x^4 - x^2 y + \frac{3}{4}y^2 - 2y + 5$$

and determine all relative maximum, relative minimum, and saddle points.

Question 7 Use Lagrange multipliers to find the maximum and minimum values of $f(x,y,z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$.

Question 8 Evaluate the following double integral:

$$\int_0^2 \int_{y/2}^1 y e^{x^3} \, dx \, dy$$

Question 9 Find the volume of the solid in space which lies below the surface $z = 3 + \cos y$ and above the region in the xy–plane bounded by the curves $x = \pi$, $y = 0$, and $y = 2x$ by evaluating an appropriate double integral.

Question 10 Find the volume determined by $z \leq 6 - x^2 - y^2$ and $z \geq \sqrt{x^2 + y^2}$.